Coherence of Pulsed Microwave Signals Carried by Two-Frequency Solid-State Lasers

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***Abstract*—This paper studies the coherence of the radio-frequency (RF) beat note between the two frequencies emitted by a two-eigenstate solid-state laser operating in pulsed regime. Depending on how the pulsed operation regime is obtained—passive Q switching using a saturable absorber or resonant amplitude modulation using pump-power modulation—we show theoretically and experimentally that the RF beat note loses or maintains its coherence from pulse to pulse. Namely, RF coherence is shown to be lost when the laser intensity vanishes between pulses, while it can be maintained if a slight amount of intensity remains inside the cavity between pulses. We give experimental demonstrations of these results for both Nd:YAG and Er-Yb:glass two-frequency pulsed lasers, in connection with applications to lidar-radar systems.**

***Index Terms*—Microwave generation, photon statistics and coherence theory, relaxation oscillations and long pulse operation, two-frequency lasers.**

I. Introduction

COMPACT all solid-state laser sources are now commonplace in the laser field, either in continuous wave (CW) or Q-switched regimes [1], [2]. These lasers are widely used in scientific, medical, industrial, and military systems [3], [4]. Besides, there is a growing interest in beat-note-carrying optical sources, whose potential applications include radio-on-fiber [5], [6] and lidar-radars [7], [8], for example. Recently, it has been shown that solid-state two-frequency lasers are attractive in terms of beat note tunability and spectral purity [6], [9], [10]. In particular, dual-polarization operation of passively Q-switched lasers has been shown to provide a suitable means for generating pulsed adjustable beat notes [11], [12] that are useful for applications using the lidar-radar concept [7]. In the framework of this application, range and Doppler resolution critically rely on the coherence time of the microwave signal, which must be much longer than the optical pulse duration [8]. Unfortunately, Q-switching usually leads to a complete extinction of the laser intensity between pulses. As a result, the phase of the microwave signal is expected to be lost, limiting the temporal window of lidar-radar signal processing to the duration of one pulse only. On the one hand, since CW dual-frequency oscillators offer high-purity beat notes, one could use a master CW dual-frequency oscillator followed by a pulsed amplifier, or use the technique of injection seeding [13], [14] in the dual-frequency Q-switched oscillator, in order to improve the coherence time of the pulsed microwave signal. But these schemes are quite complex. On the other hand, it is long known that solid-state lasers can be operated in pulsed regime by modulating their pump power at frequencies close to their relaxation oscillation frequency [15], [16]. Namely, pump-power modulation can produce either the so-called spiking regime, where the laser intensity drops off between pulses, or the resonant amplitude-modulated (AM) regime, in which the laser intensity does not vanish completely between pulses [17], [18]. One can hence wonder whether modulating the pump power of a dual-frequency laser could provide a means to make the laser oscillate in a pulsed regime while keeping the beat note coherent from pulse to pulse. The aim of this paper is to study theoretically and experimentally the simple pump-power modulation technique of a diode-pumped dual-frequency solid-state laser and compare it with passive Q switching. In particular, we wish to investigate the conditions to obtain two-frequency laser pulses. In order to study the pulse-to-pulse coherence, we analyze the resulting microwave spectrum.

To this aim, we first calculate in Section II the microwave power spectral density of a train of model two-frequency pulses. This calculus allows us to show the influence of coherence from pulse to pulse on the beat note spectrum. The expected spectrum is then compared to the experimental one obtained from a usual Q-switched laser. In Section III, we compute the laser dynamics using a rate equations model with modulated pump power. The predictions provided by this analysis are then compared in Section IV with experimental results obtained on a Nd:YAG laser emitting at 1064 nm and an Er-Yb:glass laser emitting at 1550 nm. The conditions to obtain a coherent beat note are isolated. Finally, the results are summarized and discussed in Section V.

II. Spectrum of a Model Pulsed Two-Frequency Laser

We aim here at showing that a signature of the pulse-to-pulse coherence is obtained from the RF spectrum analysis. In order to

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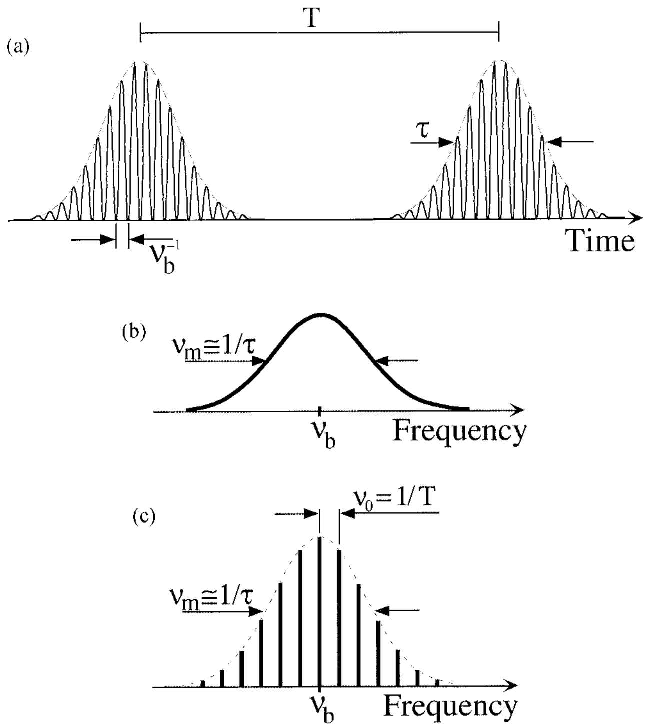

Fig. 1. (a) Sketch of a train of optical pulses (duration τ, period T) carrying a beat note at microwave frequency ν_b. Corresponding Fourier transform in the case of (b) an incoherent beat note and (c) a fully coherent beat note.

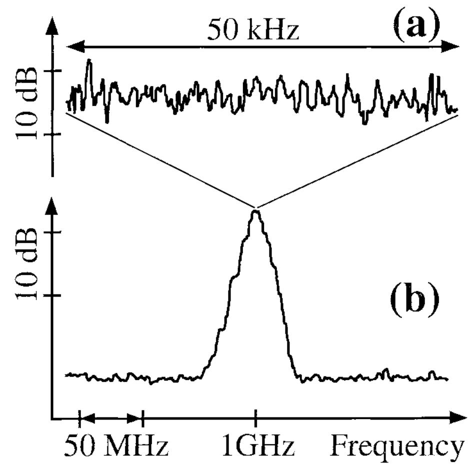

Fig. 2. RF spectrum of the pulse train emitted by a two-frequency Nd^{3+}:YAG laser passively Q-switched by Cr^{4+}:YAG, as in [8]. Pulse duration: 36 ns, repetition rate: 4 kHz, and beat note frequency: 1.0 GHz. (a) Zoom on the beat note center frequency, span 50 kHz, RBW 1 kHz. Note the flat spectrum, showing the lack of pulse-to-pulse coherence. (b) Span 400 MHz, RBW 1 MHz.

predict the spectral features that one can expect from the measurement of an optically carried pulsed microwave signal, we first consider here a train of two-frequency Gaussian pulses, such as the one shown in Fig. 1(a). When detected by a photodiode, the ac photocurrent generated by such an optical pulse train can be given by the following model analytical expression:

$$I(t) = \left\{ I_0 \exp\left(-\frac{t^2}{\left(\frac{\tau}{2\sqrt{\ln 2}}\right)^2} \right) \otimes \sum_n \delta(nT - t) \right\} \cdot \cos\left(2\pi\nu_b t + \varphi(t)\right). \quad (1)$$

The brackets enclose the convolution product (denoted by $\otimes$) of a Gaussian pulse with a peak power I_0 and a time duration τ (full-width at half-maximum) with a comb of Dirac functions with a period T (n is an integer). This usual pulse train is here modified by the modulation term at the beat note frequency ν_b (i.e., the microwave frequency to be detected) with a phase $\varphi(t)$. In order to predict the experimental signature of the resulting beat note coherence, we calculate the power spectral density of $I(t)$. It is given by the Fourier transform of the autocorrelation function of $I(t)$ [19]. Assuming that the phase can be described by a stationary process with a power spectral density $S_\varphi(\nu)$, a simple calculation yields the pulse train electrical power spectral density $S_I(\nu)$ around the beat frequency

$$S_I(\nu) = \left\{ S_0 \exp\left(-\frac{\nu^2}{\left(\frac{\nu_m}{2\sqrt{\ln 2}}\right)^2} \right) \cdot \sum_n \delta(n\nu_0 - \nu) \right\} \otimes S_\varphi(\nu - \nu_b). \quad (2)$$

Hence the frequency comb with a period $\nu_0 = 1/T$ is contained in a Gaussian envelope with an amplitude S_0 and a spectral width ν_m [full-width at half-maximum (FWHM)], the latter being linked to the pulse duration through $\tau \times \nu_m = 0.62$. The spectrum is given by the convolution product of this frequency comb with the power spectral density of the beat note. Let us analyze this expression in two opposite cases. First, considering the "incoherent" case, corresponding to a random $\varphi(t)$ from pulse to pulse (with a correlation time lower than T), we predict the microwave spectrum to exhibit a smooth Gaussian shape centered on the beat frequency, with a width on the order of ν_m, as depicted in Fig. 1(b). Second, when we consider the "coherent" case, corresponding to a constant deterministic phase $\varphi(t) = \varphi_0$, hence $S_\varphi(\nu - \nu_b) = \delta(\nu - \nu_b)$, then we predict the microwave spectrum to exhibit a ν_0-period comb around the beat frequency, within the Gaussian envelope of width ν_m, as depicted in Fig. 1(c). In the intermediate case (partial coherence among pulses), each component of the frequency comb is simply broadened by the beat note phase noise. In view of these predictions, we investigate a first experimental case.

Q switching a two-frequency solid-state laser with a passive saturable absorber is an efficient technique to get an optically carried pulsed microwave signal [11]. Let us briefly describe the beat note spectrum obtained with such a laser. For example, we consider a typical diode-pumped Nd:YAG laser with a specially designed Cr:YAG saturable absorber, which ensures two-frequency operation (for details, see [8]). It emits a train of 36-ns-long pulses with a 250-μs period. Two optical frequencies at around 282 THz (1064-nm wavelength) are emitted simultaneously in each pulse. The beat note frequency is continuously adjustable between 0 and 2.65 GHz by varying an intracavity phase anisotropy. We choose a 1.0-GHz beat frequency and observe the power spectrum after detection of the pulse train by a fast photodiode. The signal is reproduced in Fig. 2(b). The spectrum envelope width is about 20 MHz, yielding a time–bandwidth product $\tau \times \nu_m \cong 0.7$. From the analysis above, we know that coherent pulses would create a frequency comb inside this envelope with a period $\nu_0 = 4$ kHz. However, as depicted in

Fig. 2(a) with a 1-kHz resolution bandwidth, no internal comb structure is present inside this broad spectrum. Note that the repetition rate was stabilized to better than 32 mHz [20], ensuring that the phase noise comes from the beat frequency only. This result shows that two-frequency Q-switched pulses can generate a microwave signal whose coherence time is limited to the duration of one pulse only. This can be understood by the fact that the laser switches off between pulses and loses its spectral memory [21]. At each new pulse, the two optical modes start with their own random phases. The resulting microwave beat note hence has a random pulse-to-pulse phase. In order to circumvent this drawback of Q switching, we now turn to another technique to make our two-frequency laser emit pulses while maintaining a nonzero intensity between pulses.

III. Modulation of the Pump Power: Theory

In order to investigate the potentialities of pump-power modulation on a solid-state laser for our purpose, we simulate a simple Nd:YAG laser by integrating rate equations with a modulated pump parameter, as discussed in [17], [18], and [22]–[25]. To describe the time evolution of the laser intensity in the presence of this pump modulation, we neglect the existence of the two eigenstates and treat it as purely scalar. Namely, the laser is described by the following set of equations of evolution for the single-frequency laser intensity and populations in the active medium:

$$\frac{dI}{dt} = -\Gamma I + \kappa(n_u - n_d)I + \kappa n_u \varepsilon \tag{3a}$$

$$\frac{dn_u}{dt} = \gamma_u (P(t) - n_u) - \zeta(n_u - n_d)I - \zeta n_u \varepsilon \tag{3b}$$

$$\frac{dn_d}{dt} = \gamma_u n_u - \gamma_d n_d + \zeta(n_u - n_d)I + \zeta n_u \varepsilon \tag{3c}$$

where I is the intracavity laser intensity, n_u and n_d are the populations of the upper and lower levels of the laser transition, respectively, Γ is the cavity decay rate, κ and ζ are Nd^{3+}-atom/field coupling coefficients, ε is a small term that holds for the spontaneous emission, and γ_u and γ_d are the relaxation rates of the upper and lower laser levels, respectively. Finally, $P(t) = [\eta + \Delta\eta \sin(2\pi f t)]P_{th}$ is the modulated pumping rate normalized to the upper level decay rate. In this last expression, P_{th} is the pump power at laser threshold, η is the average excitation parameter, $\Delta\eta$ is its modulation amplitude, and f is the modulation frequency. We aim at isolating some values of parameters η, $\Delta\eta$, and f for which the laser emits pulses without switching off between pulses. The laser behavior is obtained by a numerical integration of (3) using a fourth-order Runge–Kutta algorithm [26]. In all the simulations, we take $\Gamma = 125$ MHz (corresponding to a $L = 36$ mm-long cavity with 3% losses per roundtrip), $\kappa = 1$ and $\zeta = 1$, $\varepsilon = 10^{-20}$, $\gamma_u^{-1} = 230$ μs, and $\gamma_d^{-1} = 50$ ns. The simulation is first run at fixed values of the pump power in order to derive the relaxation oscillation frequency $f_{\rm RO}$. We find $f_{\rm RO}$ values that nicely fit the standard formula $2\pi f_{\rm RO} = \sqrt{(\eta - 1)\gamma_u \Gamma}$ [27]. In order to simulate situations that we can reach experimentally, we choose $\eta = 3.65$, for which the simulation yields $f_{\rm RO} = 204$ kHz.

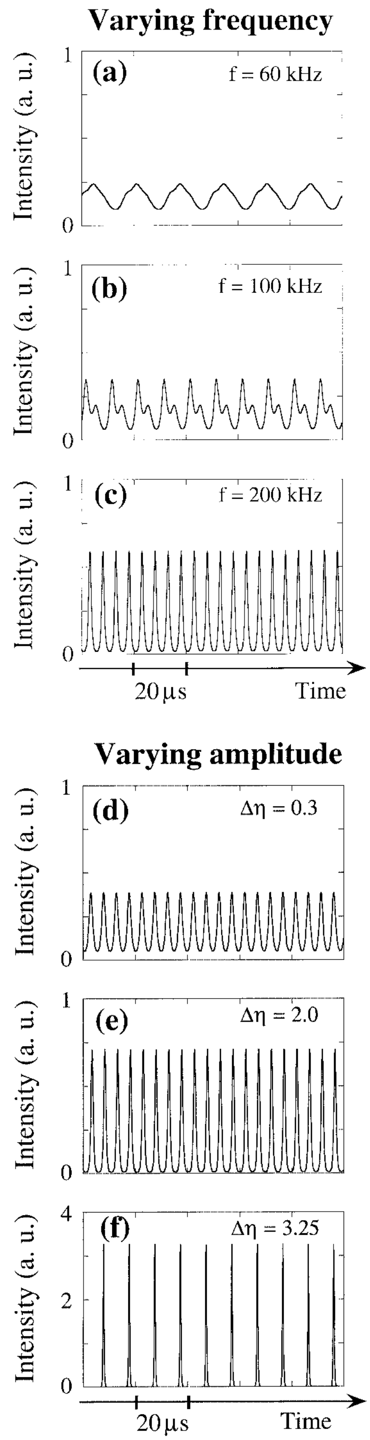

Fig. 3. Theoretical temporal evolution of the Nd:YAG laser output intensity. Equations (1)–(3) are numerically integrated using parameters given in text. Various regimes are obtained when (a)–(c) the modulation frequency is varied or (d)–(f) the modulation amplitude is varied. Note that in (f), the laser intensity drops down to zero (spiking regime), whereas in (c) or (e), the laser exhibits nice pulse shapes without switching off.

First, we fix the modulation depth at a value corresponding to $\Delta\eta = 1$ around $\eta = 3.65$. We compute the laser output intensity for different values of the modulation frequency and observe that the simulated laser output is amplitude-modulated with an increasing modulation depth when $|f - f_{\rm RO}|$ decreases. Fig. 3(a)–(c) depicts the simulated laser intensity for three different values of the modulation frequency. The resonant AM appears clearly, with a modulation depth increasing as the modulation frequency gets closer to $f = f_{\rm RO} = 204$ kHz. In particular, at this modulation frequency, the laser simulation yields a nice 1-μs-long pulse train at the modulation frequency with a nonzero intensity between pulses.

Secondly, the laser is operated at $\eta = 3.65$, and the modulation frequency is fixed at $f = f_{\rm RO} = 204$ kHz. The laser output intensity is computed for different values of the pump modulation amplitude. Fig. 3 (d)–(f) shows the results in three different situations corresponding to increasing values of $\Delta\eta$. One can note that the case $\Delta\eta = 2$ permits one here again to obtain the requested behavior for our purpose [see Fig. 3(e)], whereas the case $\Delta\eta = 3.25$ corresponds to the so-called spiking regime for which 1) the modulation amplitude is such that the pump power drops below threshold value at each period and 2) the laser switches off between the 500-ns-long pulses. Note that, in this case, the spike repetition rate is half the modulation frequency. In order to have a spike repetition rate equal to the modulation frequency, we verify numerically that one can reduce the pump parameters closer to threshold, for example, $\eta = 1.5$ and $\Delta\eta = 1$, with a lower modulation frequency, e.g., 25 kHz.

Following these theoretical predictions, we infer that the resonant AM mode, such as illustrated in Fig.3 (a)–(e), can yield a coherent microwave pulsed signal provided that the laser is operated two-frequency. Among these examples, cases depicted in Fig. 3(c) and (e) seem to offer the best solution for our purpose. We also note that a strong modulation, such as the one illustrated in Fig. 3(f), could be detrimental for the beat note coherence, because the intensity is null between pulses. We wish to test now these predictions experimentally on two different lasers.

IV. Experimental Results

We consider a longitudinally pumped two-frequency solid-state laser [28]. The resonator has a length $L = 36$ mm and is closed by two mirrors M_1 and M_2. M_2 (transmission $T = 1\%$ at 1064 nm) is a spherical mirror of 200-mm radius of curvature. M_1 is highly transmitting $(T > 95\%)$ at 808 nm and highly reflecting $(R > 99.5\%)$ at 1064 nm. It is directly coated on the active medium, which is a 1.1-mm-long crystal of 1% at doped Nd:YAG. The two orthogonal eigenstates of the laser are obtained by introducing two antireflection coated quarter-wave plates inside the cavity [8]. The frequency difference is then given by $\Delta\nu = \rho c/\pi L$, where c is the velocity of light and ρ is the angle of the fast axis of one quarter-wave plate with respect to the slow axis of the other. We choose the orientations of the quarter-wave plates to reach a 1.0-GHz beat note. The laser is pumped continuously with a fiber-coupled laser diode emitting at 808 nm. The threshold pump power is 130 mW. In these conditions, this laser corresponds to the parameters used in the simulations of Section III.

Since higher peak powers are predicted in the spiking mode, we first choose to modulate the laser pump power at a frequency of $f = 25$ kHz with $\eta = 1.5$ with a modulation depth $\Delta\eta = 1$, so that the pump power drops below threshold at each period. The corresponding experimental pulse train is shown in Fig. 4(a). As expected, the laser emits spikes with a period equal

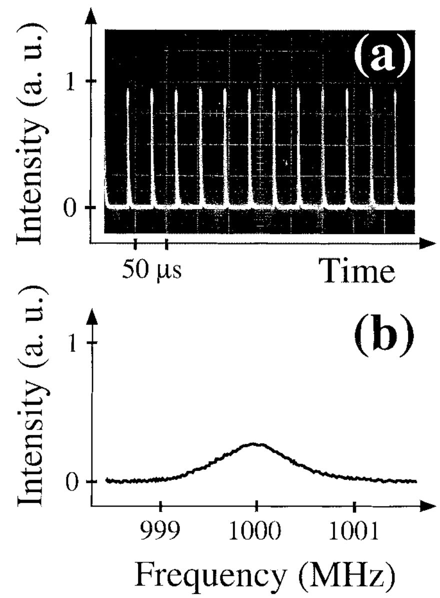

Fig. 4. (a) Experimental pulse train obtained with the Nd:YAG laser in the spiking regime. (b) Corresponding RF spectrum showing the lack of pulse-to-pulse coherence.

to the period of the pump modulation. The laser intensity drops down to zero between spikes, as also predicted by the model. We then verify that these spikes contain the beat note at 1 GHz by inserting in the output laser beam a polarizer whose axis is oriented at 45° of the laser eigenstates. Fig. 4(b) shows the experimental power spectral density around the beat frequency. We observe a Gaussian-like envelope of width roughly 800 kHz. From the theoretical predictions of Section II, we know that coherence from pulse to pulse would also result in the appearance of a comb with a 25-kHz period inside the Gaussian envelope. Unfortunately, this is not the case here. Similarly to the Q-switched laser seen in Section II, the spiking mode here leads to a poor beat note coherence due to the loss of spectral memory of the laser between spikes. A similar phenomenon was predicted and observed in the study of the spiking mode at laser onset [21]. It was shown that when the photon number drops close to one per mode after one spike, the spectral memory is lost. As a result, the next spike grows with a new random mode number, and, *a fortiori*, a random phase.

The incident pump power is now raised up to 645 mW so that the measured relaxation oscillation frequency is 204 kHz. At this point, we measure a 45-mW output power. Moreover, when the modulation frequency is raised up to $f = 204$ kHz and the pump modulation amplitude is set at $\Delta\eta = 1.5$, the resonant AM mode is observed experimentally. Namely, quasi-symmetrical pulses with a pulse duration of 900-ns FWHM at a repetition rate of 204 kHz are emitted, as seen in Fig. 5(a). The peak power is about five times as high as the average CW power, and the intensity does not drop down completely to zero between pulses (modulation depth is 93%). The microwave spectrum is then measured around 1 GHz. The experimental result is reproduced in Fig. 5(b) in linear scale. The comb structure with the 204-kHz period is now clearly resolved. The envelope FWHM is roughly 750 kHz, giving a time–bandwidth product of $\tau \times \nu_m \cong 0.7$. Clearly, the appearance of the frequency comb

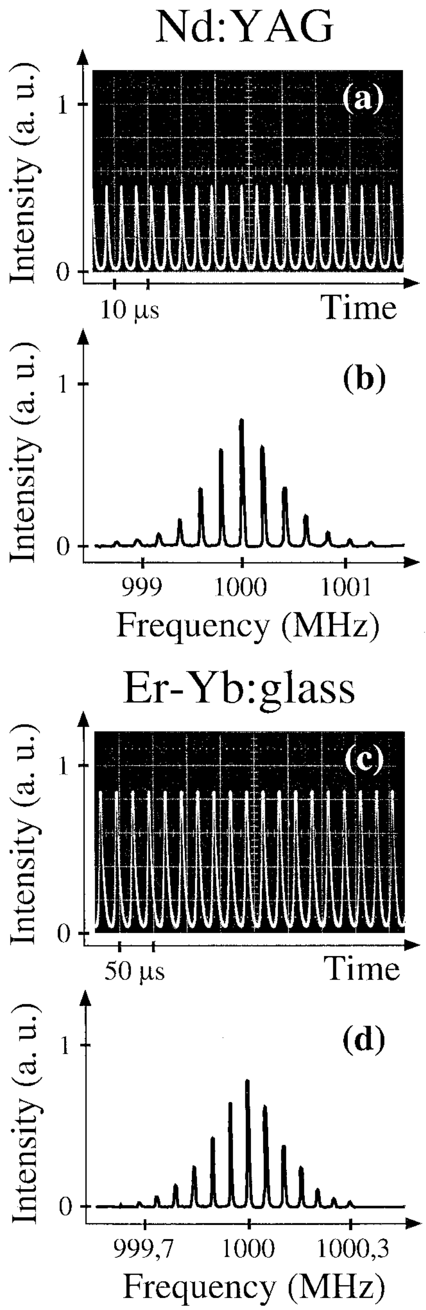

Fig. 5. Experimental pulse trains obtained in the resonant AM regime and corresponding RF spectra showing the full pulse-to-pulse coherence. (a), (b) Nd:YAG laser. (c), (d) Er-Yb:glass laser.

is the signature of the expected pulse-to-pulse coherence. As opposed to the spiking mode, the photon number between pulses is here much higher than one, hence preserving the phase memory. Since this is true for both modes, the resulting microwave phase is kept from pulse to pulse. Moreover, this regime persists for a range of modulation frequencies above 190 kHz, or below 180 kHz, with a varying modulation depth in agreement with the simulation. For modulation frequencies in between (just below the relaxation oscillation frequency), the pulse amplitude becomes unstable, and the beat note quality is degraded accordingly. The other regimes predicted and showed in Fig. 3 are also observed.

In order to check the possibility of realizing such a pulsed coherent microwave source on a 1.5-μm optical carrier, we build the same kind of laser resonator but using an Er-Yb:glass for the active medium [4], [6]. It is pumped by a fiber-coupled laser diode emitting at 975 nm. The cavity length is 50 mm. At a pump power of 260 mW (approximately $\eta = 2.3$), the relaxation oscillation frequency is $f_{\mathrm{RO}} = 56$ kHz. We choose to modulate the pump power at a frequency $f = 56$ kHz and an amplitude $\Delta\eta = 1.4$. With the simple pump modulation technique, we find the resonant AM to yield an optical pulse train at 1.54 μm carrying a coherent beat note at 1 GHz, as shown in Fig. 5 (c) and (d). The pulse duration is 3 μs and the spectral envelope FWHM is about 230 kHz, giving a time–bandwidth product of $\tau \times \nu_m \cong 0.7$. Here again, we can notice the nonzero intensity between pulses, yielding a coherent beat note [see Fig. 5(d)]. Note that 3-μs pulses could also be obtained, with a pulse peak power more than six times the average power. All these experimental results are in good agreement with the simulations. It appears that the in-phase oscillation of the two eigenstates is equivalently described by the simplified single-mode theoretical model.

Finally, in all the preceding experiments, the frequency stability of the beat note itself was comparable to the one of free-running CW two-frequency lasers, i.e., on the order of a few kilohertz, due to, e.g., spurious mechanical vibrations. Besides, phase-locked dual-frequency lasers show high-spectral purity beat notes, down to the millihertz level [10]. We hence implemented the simple pump modulation technique on such a dual-frequency laser phase-locked to an RF local oscillator. Remarkably, the experiment demonstrates a pulsed microwave signal exhibiting the frequency stability of the RF local oscillator.

V. Conclusion

We have shown theoretically and experimentally that modulating the pump power of a two-frequency diode-pumped solid-state laser gives a simple and efficient means to generate an optically carried pulsed coherent microwave signal. We have first shown experimentally that a usual Q-switched laser does not produce pulse-to-pulse microwave coherence because the laser intensity switches off between pulses. Using a common rate-equations model, we have then computed the behavior of a pump-power modulated laser. Spiking and resonant AM regimes were isolated. Experiments achieved on both a Nd:YAG laser emitting at 1064 nm and an Er-Yb:glass laser emitting at 1535 nm show good agreement with the theoretical predictions. Moreover, in the case of amplitude modulation close to the relaxation oscillation frequency, it appears that the laser output intensity is strongly modulated. Namely, less than 1-μs-long pulses with a peak power up to six times the average power are obtained. Tunable two-frequency oscillation is evidenced, resulting in an original coherent pulsed microwave beat note. Such a beat note could find applications in lidar-radar: theoretical results show that one can reach the 30-km range with pulses having the characteristics obtained here (100 ns–1 μs duration pulses at 1.55 μm carrying a beat note at 1–10 GHz, with a pulse repetition rate in the 10-μs range, and a 1-ms coherent integration time) [8]. Finally, this technique is simple and robust and can be applied to any other kind of two-frequency solid-state laser.

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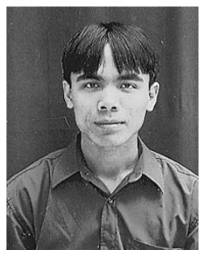

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